this time: exp. design; probability

next time: ↓

read: DD ch. 9

AMS 7 13 oct 09

(1)

hwk 2 due Thu 20 Oct 5pm

lab 2 due Fri 23 oct (BE 135)

a design is valid if it's unbiased: if you were to repeat design many times & average the results, on average the truth would emerge

matched pairs special case of randomized blocks with blocksize 2

probability

Pascal, Fermat (1650); Bayes (1720s) (1750)

pop

sample

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

ELM ✓

$P(Y_1 = 9) = \frac{1}{3}$

$P(Y_1 \text{ odd}) = \frac{2}{3}$

p(1 or more T-S kids) =

p(exactly 1 T-S kid or exactly 2 or ... or exactly 5)

p(A or B) $\stackrel{?}{=}$ p(A) ? p(B)

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p(1 or more T-S kids)

opposite is not (1 or more)

= 0 T-S kids

p(A) ?. p(not A)

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p(0 T-S kids) = p(not T-S on 1st and not T-S on 2nd and ... and not T-S 5th)

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p(A and B) $\stackrel{?}{=}$ p(A) ?. p(B)

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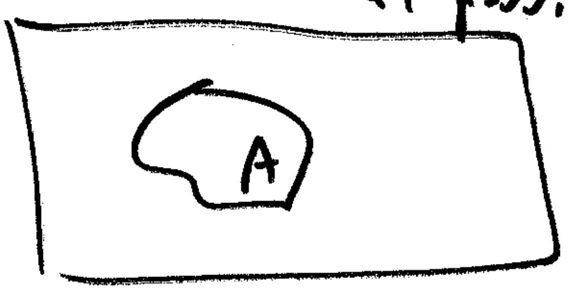
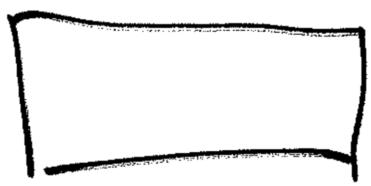
all possibilities



area of # blobs for A

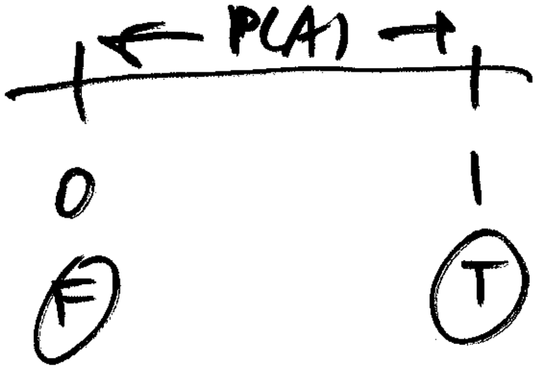
p(A) =

100% = 1 →

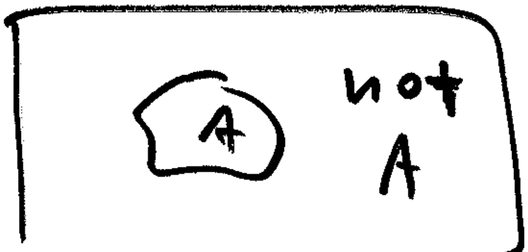


if A is certain $P(A) = 1 = 100\%$

impossible $P(A) = 0 = 0\%$



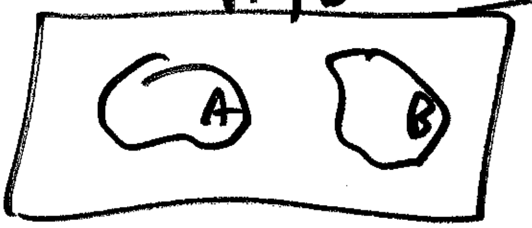
$0 \le P(A) \le 1$



$P(A) + P(\text{not } A) = 1$

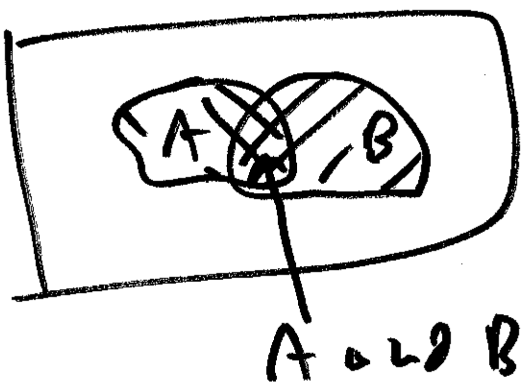
so $P(A) = 1 - P(\text{not } A)$ useful

A, B mutually exclusive (no overlap)



$P(A \text{ or } B) = P(A) + P(B)$

special case with no overlap



$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

general addition rule for or

and

$$\begin{bmatrix}1\\2\\9\end{bmatrix} \xrightarrow{\text{at random}} \begin{bmatrix}Y_1\\Y_2\end{bmatrix}$$

$P(Y_1 = 2 \text{ and } Y_2 = 2) = ?$ (*)

at random with replacement : independent identically distributed (IID)

without : simple random sampling (SRS)

IID

$$\begin{bmatrix}1\\2\\9\end{bmatrix} \xrightarrow{\text{IID}} \begin{bmatrix}Y_1\\Y_2\end{bmatrix}$$ (*) ?

joint distribution of Y_1 & Y_2

Y_1 \ Y_2	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)

ELM ✓

$$P(Y_1 = 2 \text{ and } Y_2 = 2) = \frac{1}{9}$$

$$P(Y_1 = 2) = \frac{1}{3} = \frac{3}{9} \quad P(Y_2 = 2) = \frac{1}{3} = \frac{3}{9}$$

$$P(Y_1 = 2 \,\&\, Y_2 = 2) = \frac{1}{9} = \underbrace{P(Y_1 = 2)}_{\frac{1}{3}} \cdot \underbrace{P(Y_2 = 2)}_{\frac{1}{3}}$$

with repl (IID)

theory: $P(A \text{ and } B) = P(A) \cdot P(B)$

SRS $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$ SRS $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

at random without repl.

ELM ✓

		y_2		
		1	2	9
y_1	1	~~(1,1)~~	(1,2)	(1,9)
	2	(2,1)	~~(2,2)~~	(2,9)
	9	(9,1)	(9,2)	~~(9,9)~~

$P(y_1 = 2 \ \& \ y_2 = 2) = \frac{0}{6} = 0$

$P(y_1 = 2) = \frac{1}{3} = \frac{2}{6}$

$P(y_2 = 2) = \frac{2}{6} = \frac{1}{3}$

$$P(y_1 = 2 \ \& \ y_2 = 2) = 0 \neq \underbrace{P(y_1 = 2)}_{\frac{1}{3}} \cdot \underbrace{P(y_2 = 2)}_{\frac{1}{3}}$$

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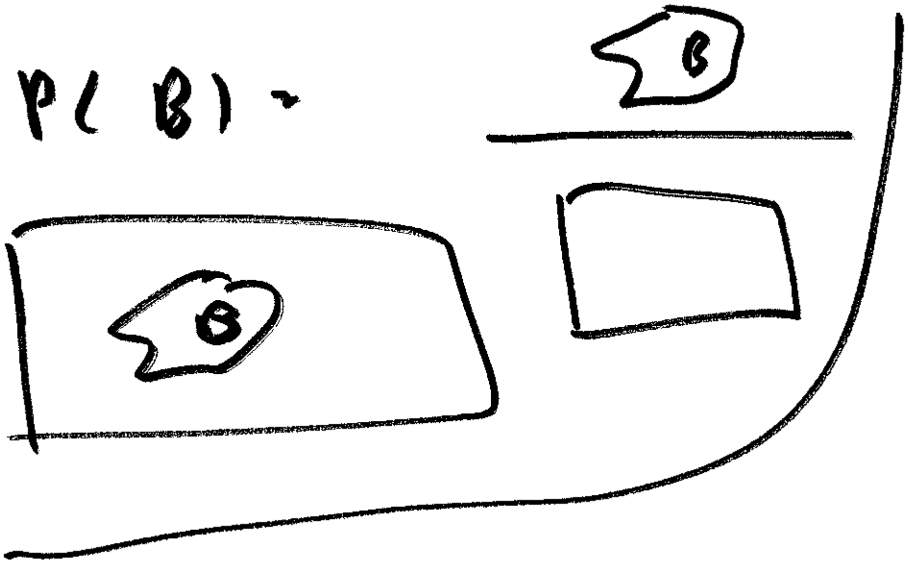
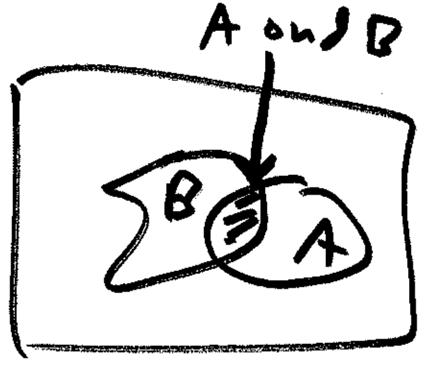
conditional probability (Bayes) (1720)

$P(B \text{ given } A) = ?$

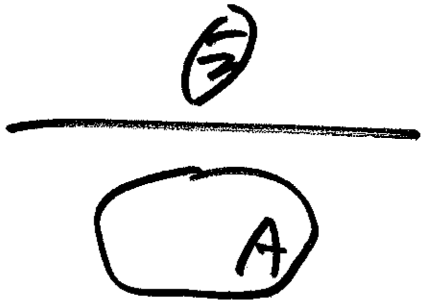
P(B) =





↓ A and B

P(B given A) =



def. :

$$P(B \text{ given } A) = P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

mult. by P(A) to get

$$= P(B) \cdot P(A \mid B)$$

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

general product rule for working with and

at random with repl.: 1st draw does not help you to predict second draw

without repl. 1st helps to predict 2nd

def. A, B independent if

knowledge about A does not

help you to predict B &

vice versa

A, B dependent

if knowledge of 1 does help

to predict the other

at random with repl: independent

identically dist. ← 2nd drawing

same prob. behavior as 1st

previous theory	if A, B indep. $P(A \text{ and } B) = P(A) \cdot P(B)$

if A, B indep $P(B \text{ given } A) = P(B)$ & $P(A \text{ given } B) = P(A)$