Brittany Guest

①

Statistical Inference

10/28

* lab 3 due date change: NOT this friday but 1 week later

* Solutions to H.W. #1 + #2 posted in glass case near Baskin 125.

Case Study: Marine Biology

$\boxed{Q}$ Does the data support the theory?

(whole) Pop — general

(probability) deductive →

sample (part) — particular

← induction (inference)

pop: all relevant crabs (ask biologist)

sample: the obs crabs

body temp @ which the equilibrate

imaginary

temp.

N = ? (big

actual like ~~SRS~~ = IID

$n = 25$

$\bar{y} = 25.0°C$

SD = 1.34

25.0
24.8
⋮

$M \to \infty$

$\mu = ?$

$\sigma = ?$

Pop. Histogram

hypothetical IID

sample histogram

normal

$n = 25$

(ex) $\bar{y} = 24.8$

⋮

long run mean = expected value of $\bar{y}$ → $E_{IID}(\bar{y}) = \mu$

long run SD = standard error of $\bar{y}$ → $SE_{IID}(\bar{y}) = \frac{\sigma}{\sqrt{n}}$

long run histo

μ

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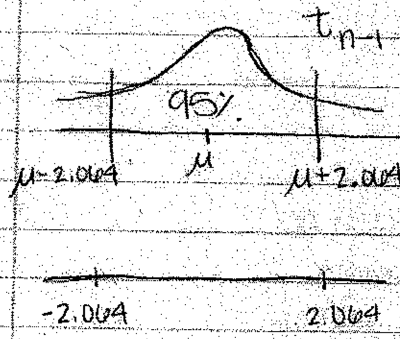
Inferential Summary

Population	unknown pop. quantity of interest	μ = pop mean ~~body~~ body temp. at which this specie of crab equilibrates to air temp (24.3°C)
sample	estimate of μ	$\bar{y} = 25.0°C$
Imaginary	give or take for $\bar{y}$ as est. of μ	$\widehat{SE}_{IID}(\bar{y}) = \frac{s}{\sqrt{n}} = 0.27°C$
Imaginary	95% CI for μ	$\bar{y} \pm 2.064\left(\frac{s}{\sqrt{n}}\right) = (24.5, 25.5)°C$

* I think μ is around 25.0°C give or take around 0.27°C.

long run histogram of $\bar{y}$, accounting for uncertainty in σ (Gossett, 1908)
*more uncertainty = heavier tails

t_{n-1} curve = t_{24}



95% of all $\bar{y}$ fall w/in ± 2.064

*Neyman (1930's): Confidence Intervals (CI)
To get an interval all the # line w/in which we are 95% confident that μ can be found, start @ $\bar{y}$ + go $t_{n-1}^{0.95}$ $\widehat{SE}$ either way.

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Formula 3 pg. 25

* Expected Value of $\bar{y}$ $\quad E_{IID}(\bar{y}) = \mu$

* Standard Error of $\bar{y}$ $\quad SE_{IID}(\bar{y}) = \frac{\sigma}{\sqrt{n}}$

* $\bar{y}$ good guess for μ
  sample histogram good guess for pop. histogram
  SD (s) good guess for σ

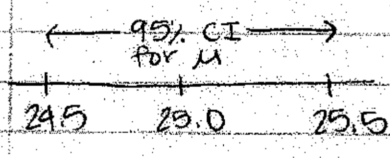
* <u>ESTIMATED</u> standard error of $\bar{y}$

$$\widehat{SE}_{IID}(y) = \frac{s}{\sqrt{n}} = \frac{1.34}{\sqrt{25}} = \boxed{0.27\,°C}$$

Formula #4

* 95% CI for M : $\bar{y} \pm t_{n-1}^{0.95} \; \widehat{SE}(\bar{y}) = \frac{s}{\sqrt{n}}$

$\bar{y}$: 25.0°C $\quad t_{n-1}^{0.95}$: 2.064 $\quad \frac{s}{\sqrt{n}}$: 0.27



$(2.064)(0.27) \approx .5$

95% confident that the true value of μ is between 24.5 and 25.5. The data does not support his theory b/c his theoretical # for $\mu = 24.3$.

* When μ_0 (theoretical value) is <u>not</u> in 95% interval, people say difference btwn μ_0 + data $\bar{y}$ is statistically significant

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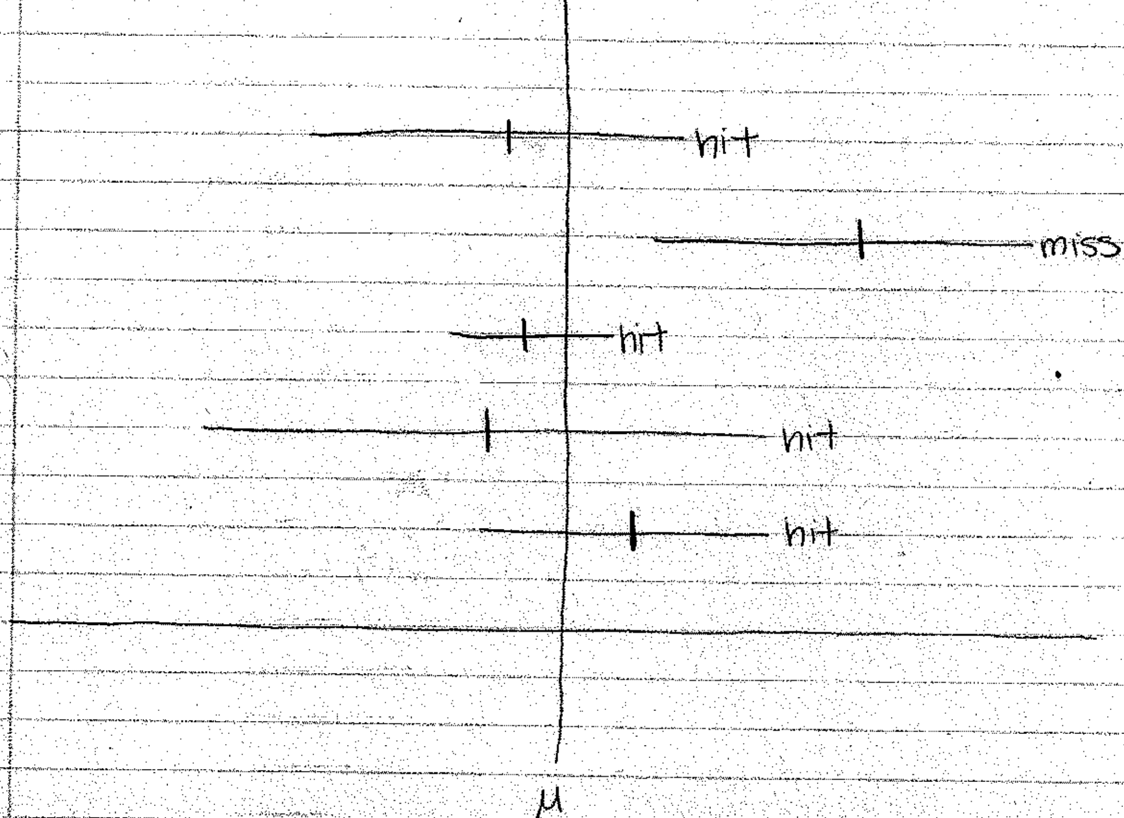
statsig (statistically significant) $\longleftrightarrow$ diff. btwn μ_0 & $\bar{y}$ hard to attribute to unlucky random sampling $\longleftrightarrow$ diff. (probably) real

Q: 95% CI for μ here is (24.5, 25.5); does this mean that the probability μ is btwn 24.5 and 25.5 is 95%?

A: Unfortunately, in frequency theory of probability, the answer is no. μ is a fixed unknown constant that either is or is not btwn 24.5 & 25.5.

95% have hits
5% have miss

*95% of these "95% CIs" should be hits



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(sample → pop.)

Inference for Proportions; hypothesis tests; Sample Size Determination 10/30

* H.W. #3 : current due date Thur. 6, may slip back
* Lab #3 due NEXT friday

* lab rats: 12 animals, 10 turned left know the food

q: Is this diff. btwen $10/12 = 83\%$ & 50% (what you expect under random L/R turning) large?

→ large in practical (biological) terms?

$1 = L$, $0 = R$ $\begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}$ mean $\frac{10}{12} = 83\%$

pop — all relevant rats = ask biologist

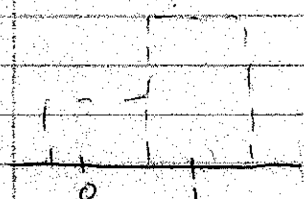
L?

N = big $\begin{bmatrix} 1s \\ \& \\ 0s \end{bmatrix}$

mean $\mu = p = ?$ (proportion)

SD σ

Pop histogram



(actual) like ~~SRS~~ = IID →

hypothetical

sample — the observed rats

L? y=1, N=0

$\begin{bmatrix} 1s \\ \& \\ 0s \end{bmatrix}$ n = 12

mean $\bar{y} = \hat{p} = \frac{10}{12} = 83\%$

$\begin{bmatrix} \vdots \end{bmatrix}$ n = 12

(ex) mean $\hat{p} = 9/12 = 75\%$

⋮

imaginary — possible $\hat{p}$

$\begin{bmatrix} 83\% \\ 75\% \\ \vdots \end{bmatrix}$ $M \to \infty$

long run mean: EV of $\hat{p} = p$

long run SD: $\widehat{SE}$ of $\hat{p} = \sqrt{\frac{0.83(0.17)}{12}}$ ← 11%

$\widehat{SE} = 11\%$

long run histo.

p

pop	unknown pop. quantity of main interest	p = pop. % of lab rats whom it is OK to generalize that would turn left
sample	estimate of p	$\hat{p} = 83\%$
imaginary	give or take for $\hat{p}$ as est of p	$\widehat{SE}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(.83)(.17)}{12}} = 11\%$
	95% CI for μ	

* EV of $\hat{p} = E_{IID}(\hat{p}) = E_{IID}(\bar{y})$

new name for $\bar{y}$

$\therefore$ EV of $\hat{p} = p$ or $E_{IID}(\hat{p}) = p$

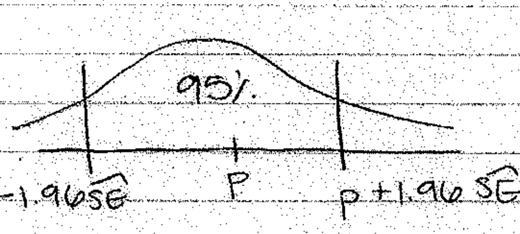
* SE of $\hat{p} = SE_{IID}(\hat{p}) = SE_{IID}(\bar{y}) = \frac{\sigma}{\sqrt{n}}$

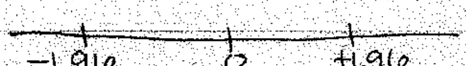
$\therefore \widehat{SE}_{IID}(\hat{p}) = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

math fact: SD σ of a population w/ mean p is:
$\sigma = \sqrt{p(1-p)}$

* long run histogram of $\hat{p}$ for ~~any~~ large $n \rightarrow$ NORMAL curve by central limit theorm

$\widehat{SE}$ 11%





standard units (Z)