Uk lottery.

49 balls

6 are drawn

chance of winning?

$$\frac{6}{49} \times \frac{5}{48} \times \frac{4}{47} \times \frac{3}{46} \times \frac{2}{45} \times \frac{1}{44}$$

$$= \frac{1}{13,983,816}$$

$\simeq$ 1 in 14 million

The size of Random Fluctuations

$p(\text{head}) = \frac{1}{2}$

$p(\text{tails}) = \frac{1}{2}$.

$$\underbrace{\#\text{heads} - \frac{1}{2}\#\text{tosses}}_{}$$

expect this to be zero

size of the fluctuation.

~~ab~~ as the # tosses increases

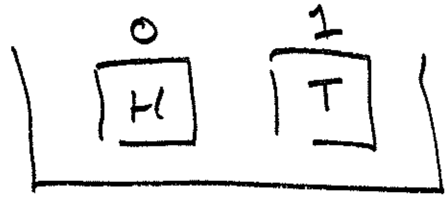
the absolute size of the fluctuations increases

size of the fluctuation
as a fraction of the # of tosses
decreases as $\frac{\sqrt{N}}{N}$

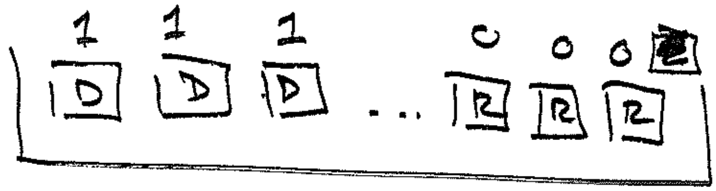
How to study these types of problems?

- analogy between the process being studied and drawing numbers at random from a box.

tossing a coin



voting.



connect the variability you want to know about with the chance variability in the SUM of the numbers drawn from the box.

$\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}$

sum of draws

= total pip count

$\boxed{0}\ \boxed{0}\ \boxed{0}\ \boxed{0}\ \boxed{0}\ \boxed{1}$ ← sum is # 6's

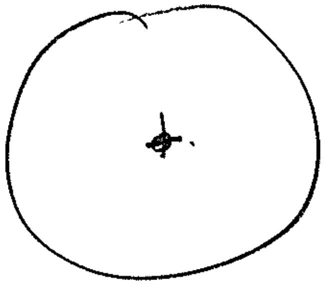
Making a Box Model.

- which numbers go on the tickets

- how many tickets of each kind

- how many draws

Roulette



38 numbered slots

1-36 ← 18 Red 18 Black.

0

00

if bet Red - get stake back plus same ammount.

on each spin gain is

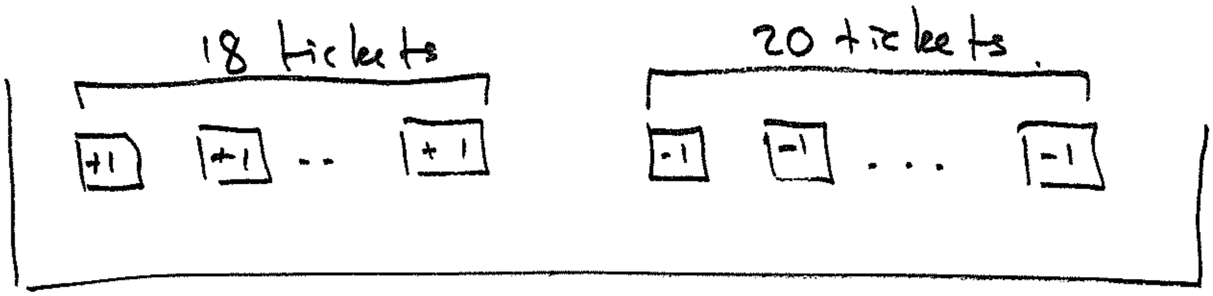
+1 (win)

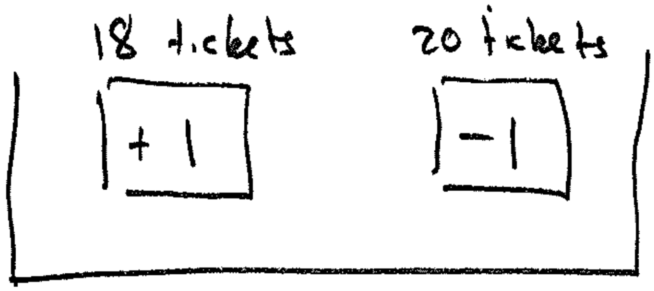
-1 (lose).

these numbers go on the tikets.

18 Red. → 18 tickets with +1

18 Black + 0 + 00 → 20 tickets with -1





Each spin is a draw with replacement

\# draws from box = # ~~time~~ of plays.

R.	R	R	B	G	R	R	B	B	R
+1	+1	+1	-1	-1	+1	+1	-1	-1	+1
1	2	3	2	1	2	3	2	1	2.

expected value

standard error ← size of fluctuations about the expected value

38 draws from the roulette box

18 times get +1

20 times get −1.

net gain $\underline{-2}$.

<u>In general</u>:

Expected value for <u>sum</u> of draws made <u>at random</u> with <u>replacement</u>

is (# draws) × (average of box).

$$38 \times \frac{18 \times 1 + 20 \times (-1)}{38} = \underline{\underline{-2}}.$$

<u>tossing a coin</u>

$$\begin{array}{cc} H & T \\ \boxed{0} & \boxed{1} \end{array}$$

average of box $= \frac{1}{2}$

sum of draws represents # tails

\# tails in 100 tosses = expected value (50) + chance variation

↑

Standard Error (SE)

SE for <u>sum</u> of draws with replacement from a box model

$$\sqrt{\#\text{draws}} \times \text{SD box.}$$

$$\text{mean of box} = \frac{0+1}{2} = \frac{1}{2}$$

$$\text{SD of box} \quad \sqrt{\frac{\left(0-\frac{1}{2}\right)^2 + \left(1-\frac{1}{2}\right)^2}{2}} = 0.5$$

100 tosses of coin

expected value is $100 \times \frac{1}{2} = 50$ tails.

$$SE = \sqrt{100} \times 0.5 = 10 \times 0.5 = \underline{\underline{5}}$$

In 100 tosses, expect 50 tails

± 5 or so.

Roulette

$$\left|\ \overset{18}{\boxed{+1}} \qquad \overset{20}{\boxed{-1}}\ \right|$$

mean of box $\dfrac{18 \times (+1) + 20 \times (-1)}{38} = -\dfrac{1}{19}$

SD of box.

$$\sqrt{\frac{\left[\left(1 - \frac{1}{19}\right)^2 + \left(1 - \frac{1}{19}\right)^2 + \left(1 - \frac{1}{19}\right)^2 + \ldots \text{ 18 times.} \quad \left(-1 - \frac{1}{19}\right)^2 + \left(-1 - \frac{1}{19}\right)^2 + \left(-1 - \frac{1}{19}\right)^2 + \ldots \text{ 20 times.}\right.}{38.}}$$

$= 0.9986$

10 spins.

expected value $\frac{-10}{19} = -0.526.$

Standard error $= \sqrt{10} \times 0.9986$

$= 3.16.$

Fluctuations are very large compared to the expected value.

Observed values are rarely more than 2 or 3 SE from the expected value.