Working Backwards.

Example : Roll a die 4 times

What are the chances of

<u>at least one</u> four ?

Think of the options

4 – – –	4 4 – –	4 4 4 –
– 4 – –	4 – 4 –	4 4 – 4
– – 4 –	4 – – 4	4 – 4 4
– – – 4	– 4 4 –	– 4 4 4
	– 4 – 4	
	– – 4 4	4 4 4 4

We want to avoid listing all these combinations

$\Rightarrow$ look at the <u>opposite</u> event.

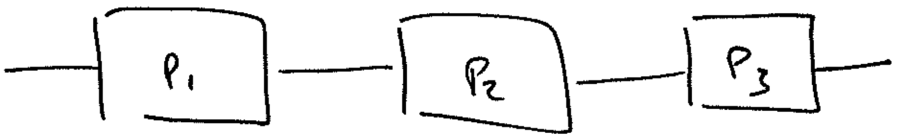
chance of <u>no</u> fours. in 4 rolls.

$= \left(\frac{5}{6}\right)^4$

Then chance of at least one four

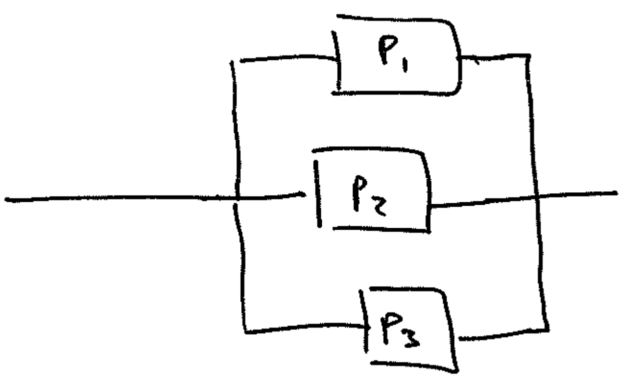
is $1 - \left(\frac{5}{6}\right)^4$

Thinking about these problems in terms of systems in <u>series</u> or <u>parallel</u>



If components are <u>independent</u> then system works if all components work,

prob $p_1 \times p_2 \times p_3$



system works if <u>any</u> component works.

$\rightarrow$ system <u>fails</u> if <u>all</u> components fail

prob fails $= (1-p_1) \times (1-p_2)(1-p_3)$.

prob. works $= 1 - (1-p_1)(1-p_2)(1-p_3)$

Summary.

event A $\qquad 0 \le P(A) \le 1$

opposite event $\qquad P(\text{not } A) = 1 - P(A)$

addition $\qquad P(A \text{ or } B) = P(A) + P(B) - \underbrace{P(A \text{ and } B)}_{=0 \text{ if mutually exclusive}}$

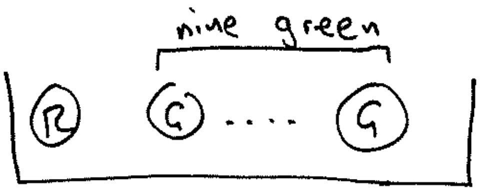
multiplication $\qquad P(A \text{ and } B) = P(A|B)\,P(B)$

$$= P(B|A)\,P(A)$$

$$= P(A) \times P(B)$$

<u>if</u> A and B are independent

Avoiding Counting.



Five draws with replacement

chance of exactly two being red.

if 2 are red, 3 are green

one possibility R R G G G

with chance $\frac{1}{10} \times \frac{1}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

Many other possibilities

R G R G G

R G G R G

R G G G R

G R R G G

G R G R G

G R G G R

⋮

10 different ways in total

Each of the 10 has the same chance:

$$\underbrace{\frac{1}{10} \times \frac{1}{10}}_{2R} \times \underbrace{\frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}}_{3G.}$$

The possibilities are <u>mutually</u> <u>exclusive</u>

=> addition rule

Add up the prob. for the 10 options.

$$10 \times \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3$$

The binomial coefficient tells us how many combinations there are binomial coefficient $\dfrac{n!}{k!\,(n-k)!}$ $\qquad \binom{n}{k}$ "n choose k"

n - # trials

k - # "successes"

The number of ways we can arrange n objects, where k are of one type, and $(n-k)$ are of the other type.

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

$1! = 1$

$0! = 1$ (~~bo~~

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

etc

success has prob p

failure $1-p$

prob of k successes in n trials

$$\frac{n!}{k!\,(n-k)!}\, p^k\,(1-p)^{n-k}$$

<u>If</u> n is fixed in advance

p is same for all trials

trials are independent

<u>Example</u> family with 4 children

what's chance of more girls than boys?

& chance of 3 girls out of 4

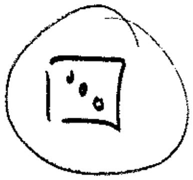
+ 4 4

$$= \frac{4!}{3!\;1!}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1$$

$$+ \frac{4!}{4!\;0!}\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^0 = 0.25 + 0.06$$

$$= \underline{\underline{0.31}}$$

Roll a die 6 times.

Chance of 4 

n - ~~no~~ # trials = 6

p - probability of success.

$p(\text{rolling } 3) = \frac{1}{6}$

k - # sucesses = 4.

chance of 4  in 6 rolls.

$$\frac{6!}{4! \times (6-4)!} \times \left(\frac{1}{6}\right)^4 \times \left(1 - \frac{1}{6}\right)^{(6-4)}.$$

$$\frac{6 \times 5 \times \cancel{4 \times 3 \times 2} \times 1}{\cancel{4 \times 3 \times 2 \times 1} \times 2 \times 1} \times \left(\frac{1}{6}\right)^4 \times \left(\frac{5}{6}\right)^2$$

$$\boxed{= 15 \times \left(\frac{1}{6}\right)^4 \times \left(\frac{5}{6}\right)^2}$$