2 sample z-test

2 sample test for RCDB trials

hypothesis tests with more than
2 classes (χ^2-test)

testing for independence

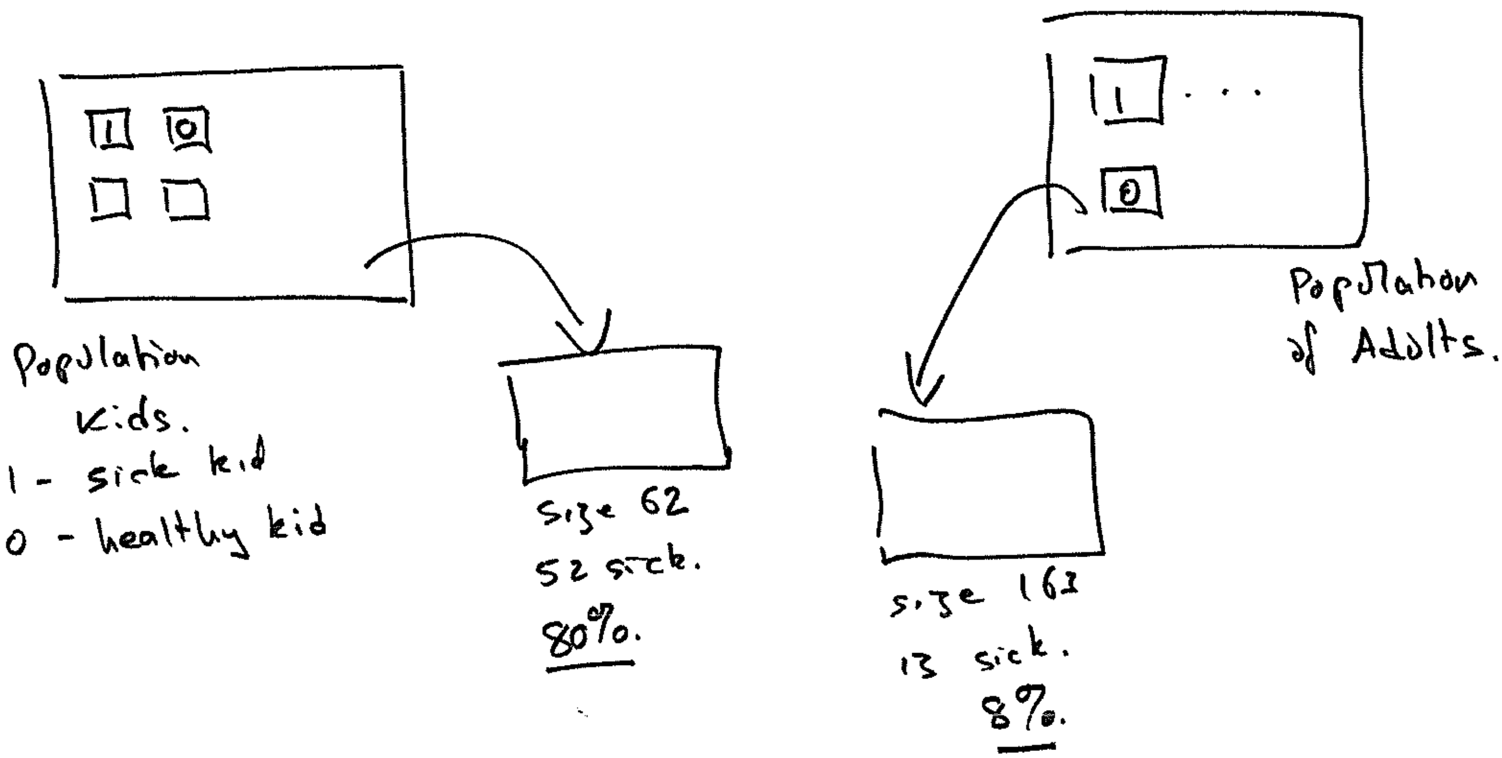
some comments on significance tests.

Flu outbreak.

Simple Random Sample of 62 kids produced 52 sick kids

S.R.S. of adults 163 adults, 13 sicks.

Q: Do kids and adults get sick at the same rate?



H_0 : % of 1's in each box is the same.

H_1 : % of 1's in each box is different.

$$z = \frac{\text{observed difference} - \text{expected difference}}{\text{SE for difference}}$$

$$= \frac{72 - 0}{\text{SE diff \%}}$$

$\boxed{\boxed{1}\ \boxed{0}}$

$$SD_{box} = \sqrt{\text{fraction of tickets with 1's} \times \text{fraction of tickets wi[th] 0's}}$$

$$SE_{\text{number of kids}} = \sqrt{62}\sqrt{0.8 \times 0.2}$$

$$SE\ \%\text{ of kids} = \frac{\sqrt{62}\sqrt{0.8 \times 0.2}}{62} \times 100 = \underline{\underline{5.1\%}}$$

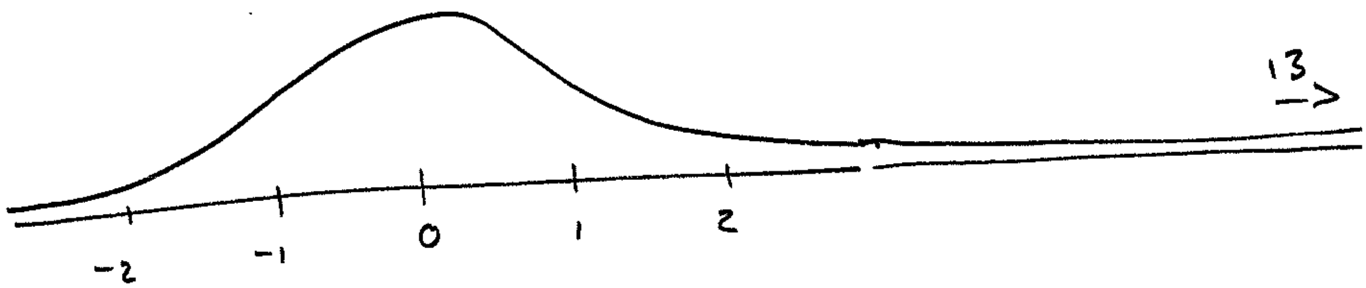
$$SE\ \#\text{adults} = \sqrt{163}\sqrt{0.08 \times 0.92}$$

$$SE\ \%\text{ adults} = \frac{\sqrt{163}\sqrt{0.08 \times 0.92}}{163} \times 100 = \underline{\underline{2.1\%}}$$

SE difference % $= \sqrt{5.1^2 + 2.1^2}$

$= 5.5\%$

$z = \frac{72}{5.5} = \underline{\underline{13}}$



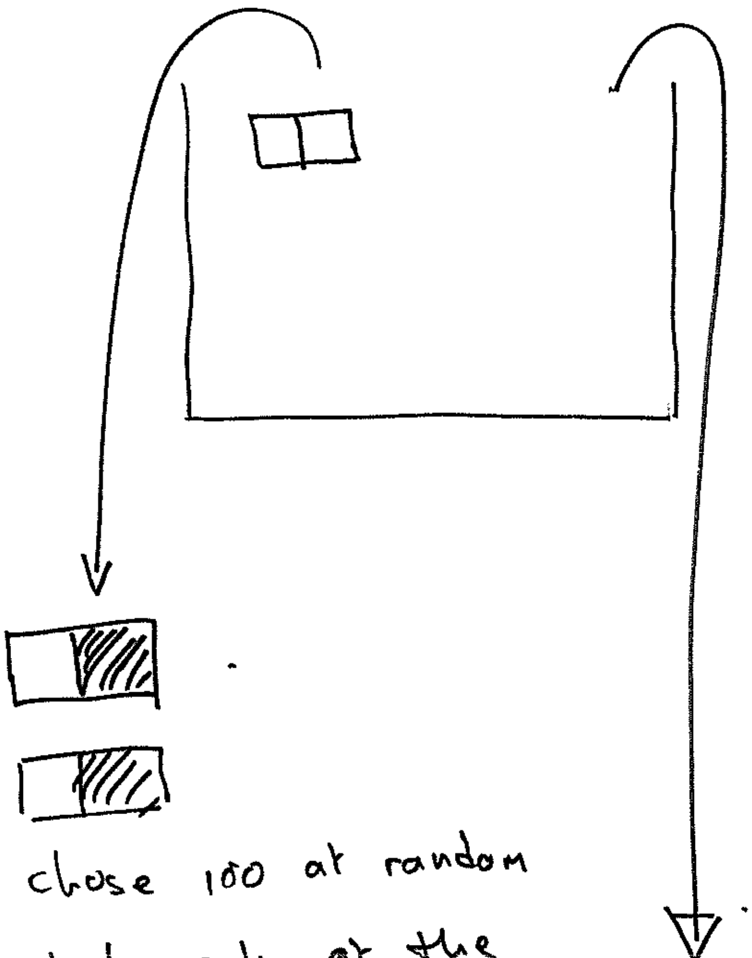
for a z-value of 3
p-value is < 1%

z = 13, p-value is << 1%.

Reject the null hypothesis; conclude that the rates of infection differ between kids and adults.

Randomized Controlled Double-Blind Trial.

- the two populations are not independent.
- but z-test can still be used.



1 ticket for each person in the trial

Two numbers on the tickets

1st: outcome if that person was in the treatment group

2nd: outcome if that person was in the control group.

chose 100 at random
- look only at the 1st number
- treatment group.

chose 100 at random

- look at 2nd number.
- control group.

Does
Vitamin C prevent colds?

treatment group.

ave # colds 2.3

SD 3.1

control group.

ave # colds 2.6

SD 2.9

Ho: average # colds is the same in the two groups

$$Z = \frac{\text{observed diff.} - \text{expected diff}}{\text{SE diff.}}$$

$$= \frac{(2.3 - 2.6) - 0}{\text{SE diff.}}$$

$$\text{SEdiff} = \sqrt{SE_1^2 + SE_2^2}.$$

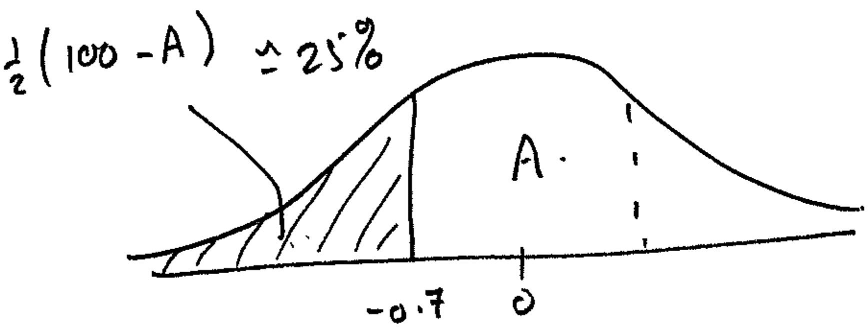
- assumes independent samples.
- sampling with replaceme

$$SE_{\text{treatment average}} = \frac{3.1}{\sqrt{100}} = 0.31$$

$$SE_{\text{control}} = \frac{2.9}{\sqrt{100}} = 0.29$$

$$SE_{\text{diff}} = \sqrt{0.31^2 + 0.29^2} \simeq 0.42$$

$$z = \frac{-0.3}{0.42} = -0.7$$



The observed difference could be due to chance

- Do not have enough evidence to reject Ho

Why can we do this ?

- draws made <u>without</u> replacement but SE computed as if draws were made with replacement. ⟹ inflates SE

- averages are dependent, but combined them as if they were independent ⟹ reduces SE

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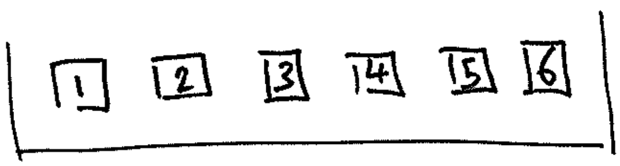
cancel.

Can we explain the data based just on random fluctuations, assuming H_0 is true?

Problems with more than 2 classes

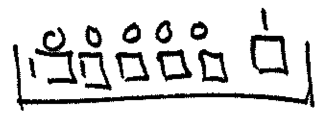
χ^2 - test.

- is a die fair?



Box model for the Null Hypothesis

For the question " is the # of 6's compatible with a fair die?"



- z test.

$$\frac{\text{obs} - \text{exp}}{SE}$$

60 rolls

$$\text{exp} = 10$$

$$SE = \sqrt{60}\sqrt{\frac{1}{6} \times \frac{5}{6}} \doteq 2.9.$$

if were to get ≤ 4 or ≥ 16 6's in 60 rolls we'd conclude die was biased

Combining all 6 numbers.

- not independent.

Need a <u>single</u> measure of how far away the observed values are from the expected values.

value	observed freq	expected freq.
1	4	10
2	6	10
3	17	10
4	16	10
5	8	10
6	9	10
	60	60

χ^2 - statistic

$$= \text{sum of } \frac{\left(\text{observed frequency} - \text{expected frequency}\right)^2}{\text{expected frequency}}.$$

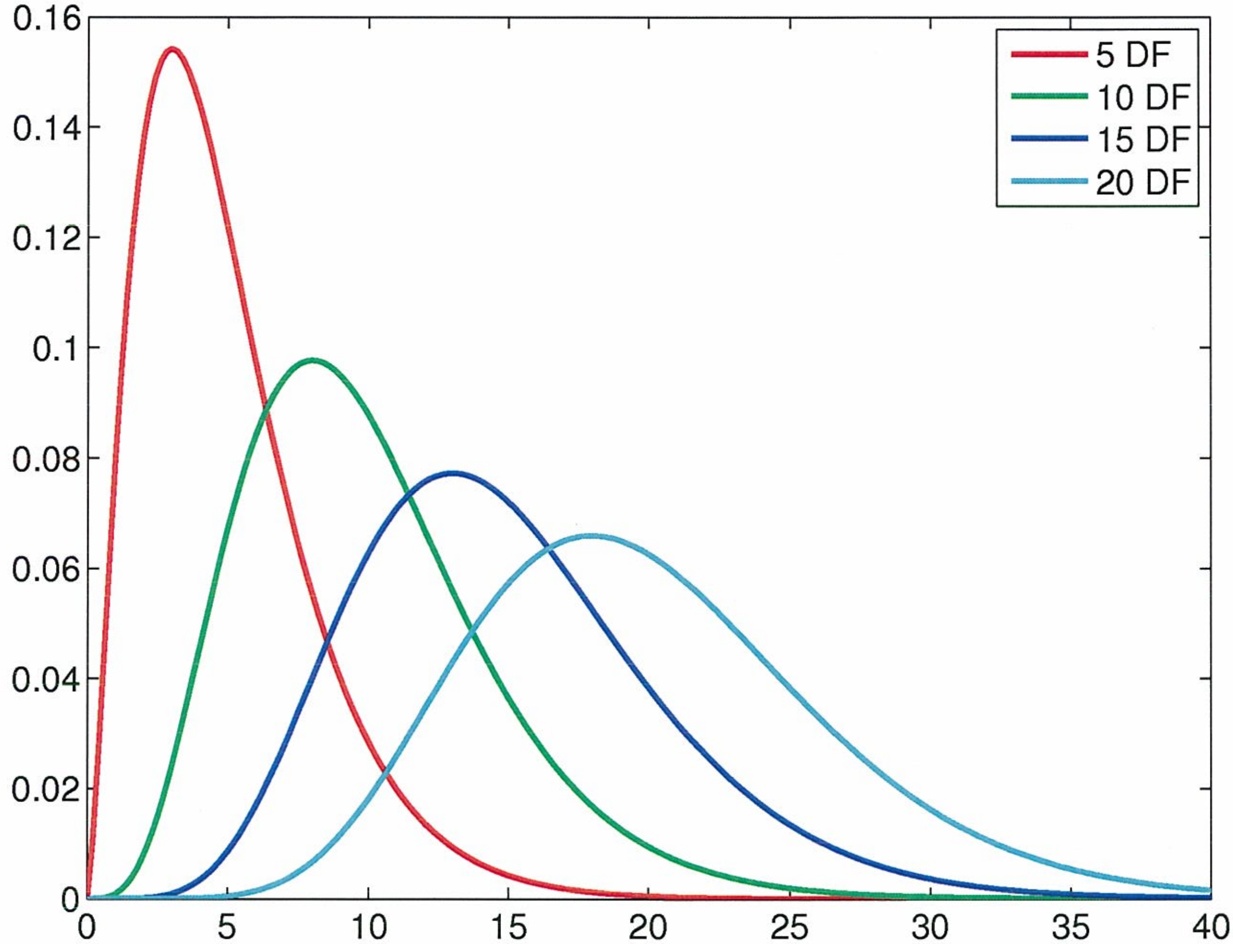
Data in the table give.

$$\chi^2 = \frac{(4-10)^2}{10} + \frac{(6-10)^2}{10} + \frac{(17-10)^2}{10} + \frac{(16-10)^2}{10}$$

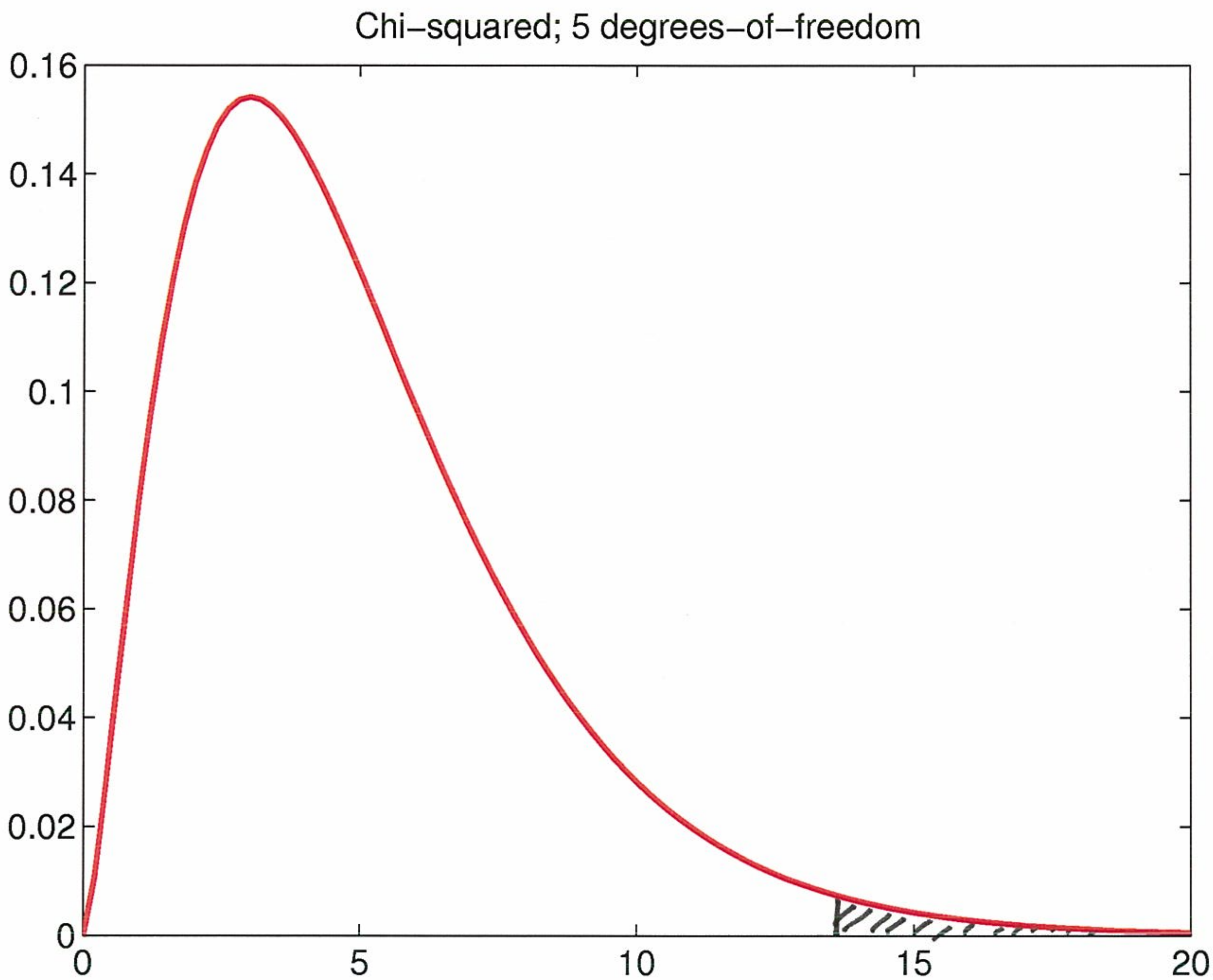
$$+ \frac{(8-10)^2}{10} + \frac{(9-10)^2}{10} = \frac{142}{10} = \underline{\underline{14.2}}.$$

<u>Now</u> If Ho is true, what's the chance of getting a χ^2 value as large as this or larger?

<u>or</u> what's the chance of rolling a die 60 times and obtaining a χ^2 value of 14.2 or greater?



5 DF
10 DF
15 DF
20 DF
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
35
40


Chi–squared; 5 degrees–of–freedom
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20

To compute the chance, we use the appropriate χ^2 - curve.

Depends on the number of <u>Degrees of Freedom</u>

Rolling die - <u>fully specified model</u>
(no adjustable parameters that we must estimate)

D.o.f = number of terms in calculation of χ^2 $- 1$.

if we know freqs. for 1, 2, 3, 4, 5 we know the freq. for 6.

$$\text{D.o.f} = 6 - 1 = 5$$

P-value : chance that we would get $\chi^2 \geq 14.2$ just on the basis of a random fluctuation.

- area to right of 14.2 on χ^2 curve with 5 DoF.

P-value corresponding to $\chi^2 = 14.2$ with 5DF.

is slightly larger than 1%.

chance of observing data as bad or worse

than the data we have is ~1%.

<u>Reject</u> H_0 . that die is fair

Notes.

1/ model is fully specified

2/ expected frequency on each row of the table is large. (≥ 5)

χ^2 - when contents of box are known

z - when average of box is known.

Summary.

a) data (observations)

b) chance model - box with known contents.

c) frequency table - observed + expected frequencies
from chance model

d) χ^2 - statistic

e) d.o.f. - # terms in χ^2 - 1

f) observed significance level - area to right of χ^2 - statistic under χ^2 curve with appropriate d.o.f.