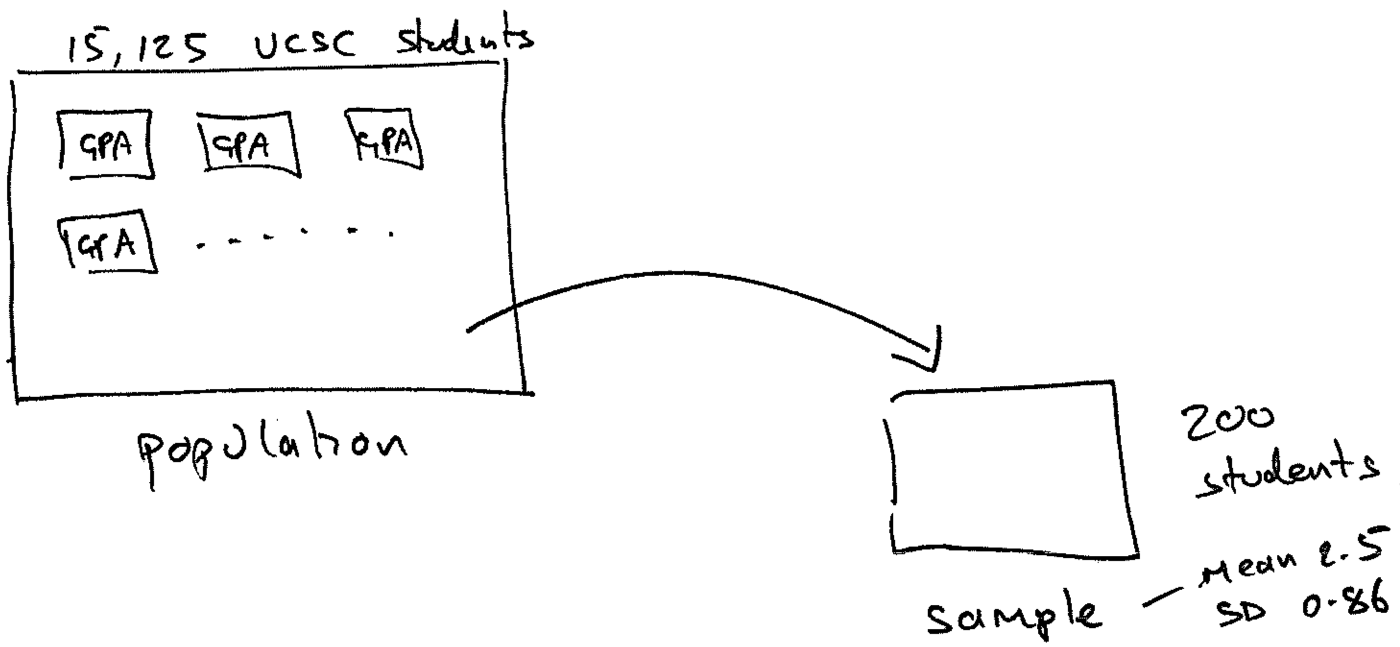
As the number of draws <u>increases</u> the sample averages become more tightly peaked around the expected value.

<u>Inference</u>.



What can I say about the <u>mean GPA</u> of all students in the population, based on my sample?

What's the average GPA at UCSC ? = average GPA of the students in the sample.

95% CI for average GPA at UCSC ?

GPAs are uniformly distributed between 1 and 4.

for my sample of 200 students

mean GPA = $\boxed{2.5}$

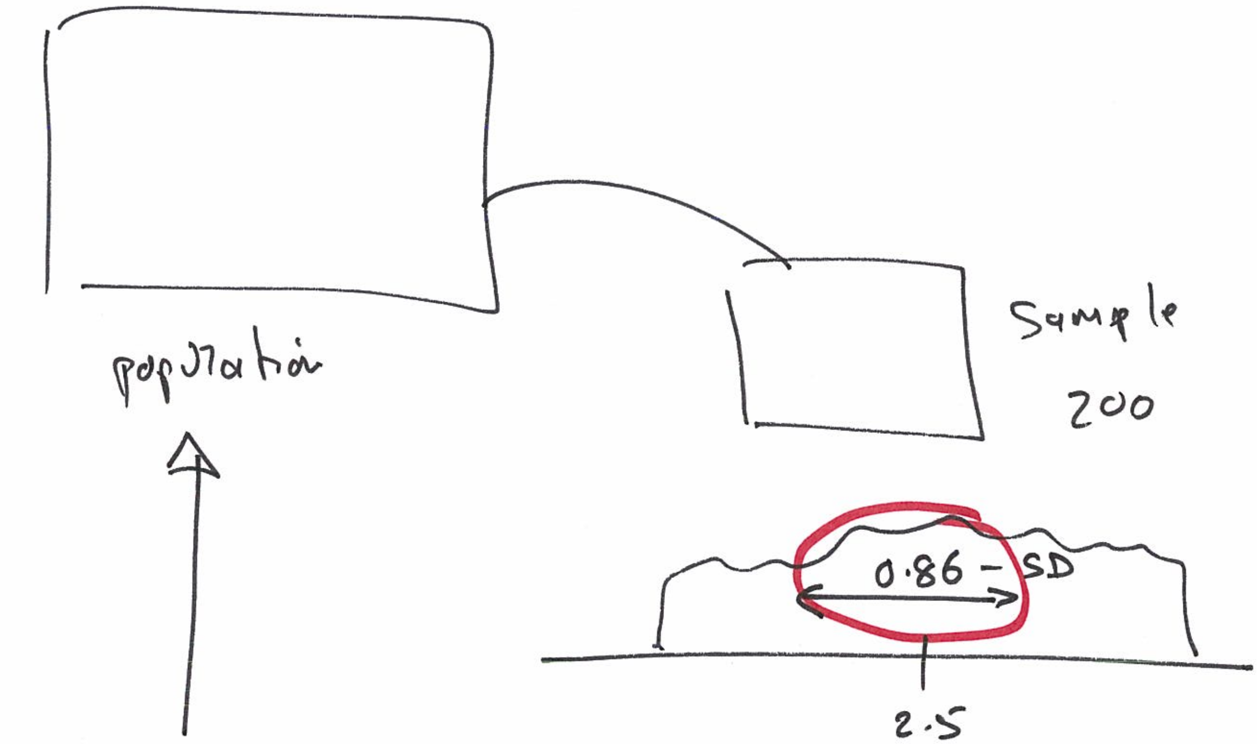
SD of sample = 0.86

SE of average = $\dfrac{\text{SD box}}{\sqrt{\# \text{draws}}} = \dfrac{0.86}{\sqrt{200}} = 0.06$

95% CI for mean GPA for population

$2.5 \pm 2 \times 0.06$

2.5 ± 0.12.



95% CI

$\boxed{2.5} \pm 0.12$.

width of CI = 0.24 ← 4 SE.

estimate of <u>mean</u>
of the population

the CI for the <u>mean</u> of the draws
is <u>much narrower</u> than the spread
in the data

Hypothesis Testing

Making decisions when uncertainty is present.

Basic question: Is the observed effect due to chance?

- use tests of significance.

Examples.

- A vaccine is known to be 25% effective. A new vaccine is being tested on a random sample of 2000 people. How do we test if the new vaccine is more effective than the old one?

- A machine fills bottles with 333 ml of liquid. Periodically a sample of bottles is taken. How to decide if the average amount is too high / too low?

Null Hypothesis vs. Alternative Hypothesis

H_0
nothing has changed
what we've observed
is due to chance

H_1
something has changed.

Vaccine: H_0 - effectiveness of new vaccine is 25%
- the same as the old vaccine (nothing has changed)

H_1 - effectiveness $> 25\%$

bottle filling : H_0 - average content is 333 ml

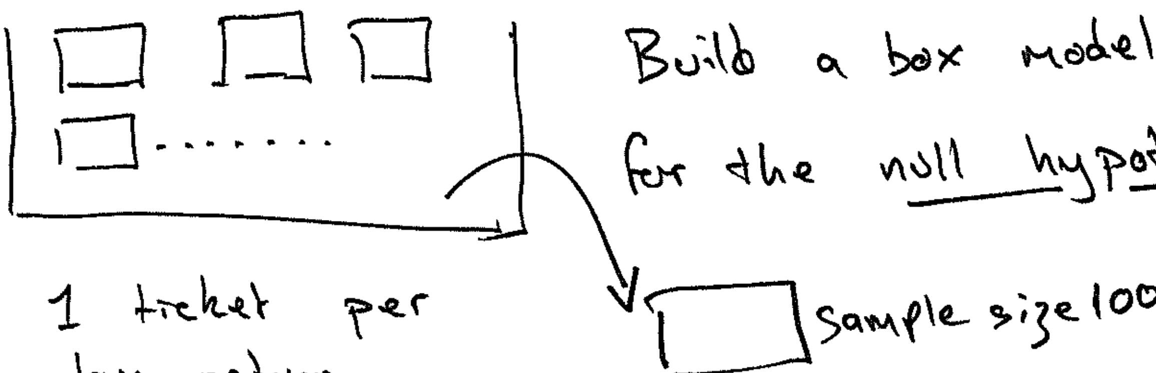
H_1 - average content is not 333 ml.

Revenue neutral changes to tax code

H_0 - revenues under new tax code are same as under the old tax code.

H_1 - revenues are different

How to test the null (H_0) against the alternative (H_1)



Build a box model for the null hypothesis (H_0)

1 ticket per tax return.

on each ticket we write the difference in tax paid under the old rules and the new rules.

Sample of 100 tax returns.

Computed old + new tax. => difference in tax paid

Average difference $-\$219$

SD of difference $\$725$

If Ho is true, expected average difference should be zero

How far away from the expected value is the observed value?

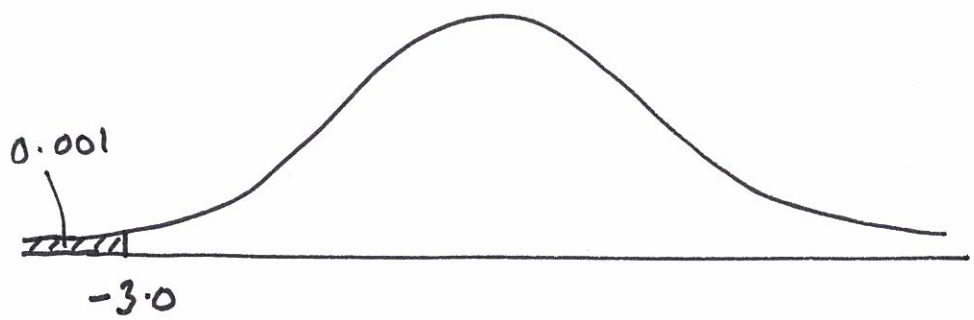
"How far away" is in terms of standard units.

$$\text{SE of } \underline{\text{average}} = \frac{\text{SD of box}}{\sqrt{\#\text{draws}}} = \frac{725}{\sqrt{100}} = 72.5$$

Compute the difference between observed and expected in standard units.

$$\frac{-219 - 0}{72.5} \simeq -3.0.$$

If the null hypothesis is true, what's the chance of getting a result as extreme as this or more extreme?



It is <u>very unlikely</u> that chance alone could result in such an extreme value.

<u>test statistic</u>. - measures the difference between data and what's expected under Ho

$$Z = \frac{\text{observed} - \underline{\text{expected}}}{SE}$$

assuming Ho is true

- how many SEs away an observed value is from it's expected value when expected value is computed using Ho observed <u>significance level</u> is the chance of getting a test statistic as extreme or more than the observed one.

This is usually denoted P, and is called the <u>P-value</u>.