Midterm

Name: ________________________________

Calculators are not allowed.

Read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. Always double-check your answers. There is some scrap paper in the back; if you need more, please ask the instructor/proctor.

Relax, and do your best!

Problem 1: Short questions. In the following questions, you are merely asked to provide the answer. No justification is needed. You should not be spending more than 1 minute per question.

1. What is the equation of the line with slope 4 and y−intercept 3? $y = 4X + 3$

2. What is the equation of the line perpendicular to $y = 2x - 1$ which goes through the point $(1, 1)$?

$m_1 \cdot m_2 = -1$, $2 \cdot m_2 = -1$, $m_2 = -1/2$ $y = -1/2\,X - b \Rightarrow b = 3/2$ $y = -\frac{1}{2}X + 3/2$ an

3. What is the center and radius of this circle? $(x + 8)^2 + (y - 5)^2 = 13$

Center: $(-8, 5)$, Radius: $\sqrt{13}$

4. Does the point $(-5, 2)$ lie on the circle of the previous question? No

Given the functions $f(x) = 2(x - 1)^2$ and $g(x) = \sqrt{3(x + 1)}$

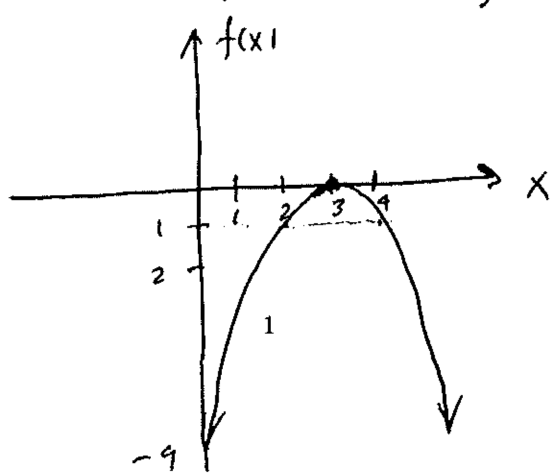
5. What is $f(2x + h)$? $f(2X+h) = 2(2X+h-1)^2$

6. What is the domain of $g(x)$? Domain $g = [-1, \infty)$

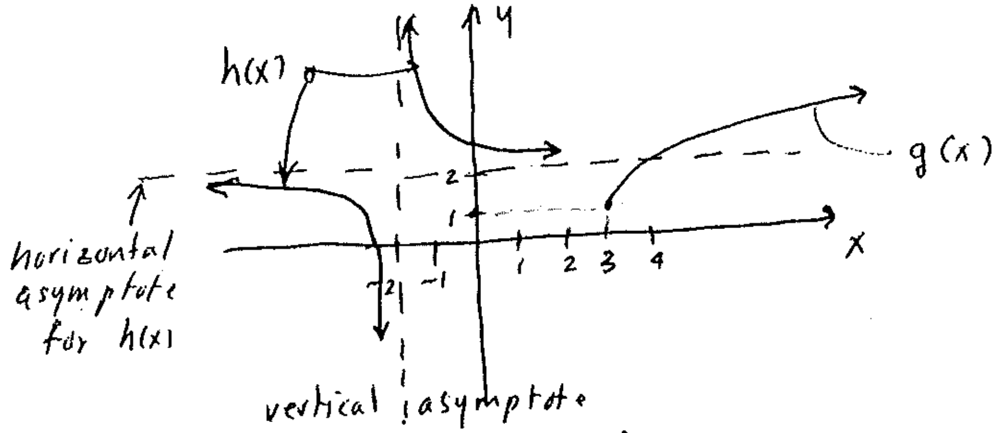
7. What is $f \circ g(x)$? $f(\sqrt{3(X+1)}) = 2(\sqrt{3(X+1)} - 1)^2$

8. What is $g \circ f(x)$? $g(2(X-1)^2) = \sqrt{3(2(X-1)^2 + 1)} = \sqrt{3(2X^2 - 4X + 3)}$

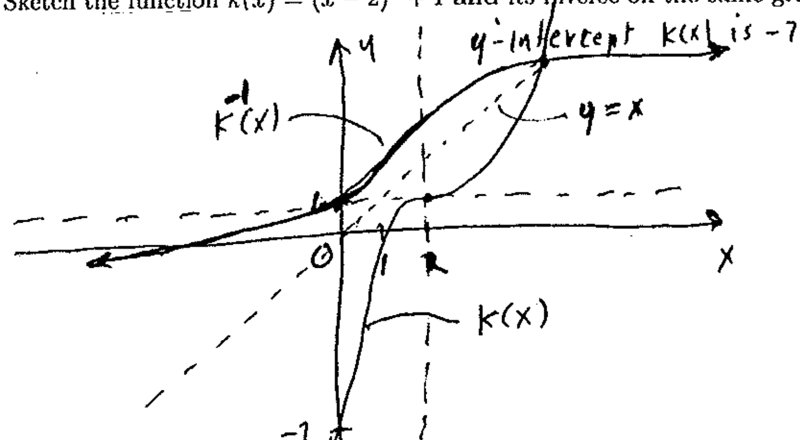
9. Sketch the function $f(x) = -(x - 3)^2$ Vertex at $(3, 0)$ y-intercept = −9



10, 11. Sketch the functions $g(x) = \sqrt{x-3} + 1$ and $h(x) = \frac{1}{x+2} + 2$



12, 13. Sketch the function $k(x) = (x-2)^3 + 1$ and its inverse on the same graph.



14. If $f(x) = x^3 + 2x + 1$, what is $f[f^{-1}(x+1)]$? ______ $x+1$ ______

15. Complete the square for the expression $-x^2+2x+3$: ___ $-(x^2-2x)+3 = -(x^2-2x+1)+4 = -(x-1)^2+4$ a ___

Given the parabola $y = 2(x+2)^2 - 4$:

16. What are the coordinates of the vertex? ______ $(-2, -4)$ ______

17. Does it open up or down? ______ opens up ______

18. What is the y-intercept? ______ $2(0+2)^2-4 = 8-4 = 4$ ans ______

19. What are the x-intercepts? ______ $-2 \pm \sqrt{2}$ Use quadratic formula ______

20. How many times does this parabola intercept the x-axis?

$f(x) = x^2 + 6x - 9$? ______ 2 times ______

PROBLEM 2: FUNCTIONS AND INVERSES: Consider the function $f(x) = \frac{x+2}{x-3}$.

(a) What is the domain of $f(x)$? Domain $f = \mathbb{R} - \{3\}$ or $(-\infty, 3) \cup (3, \infty)$

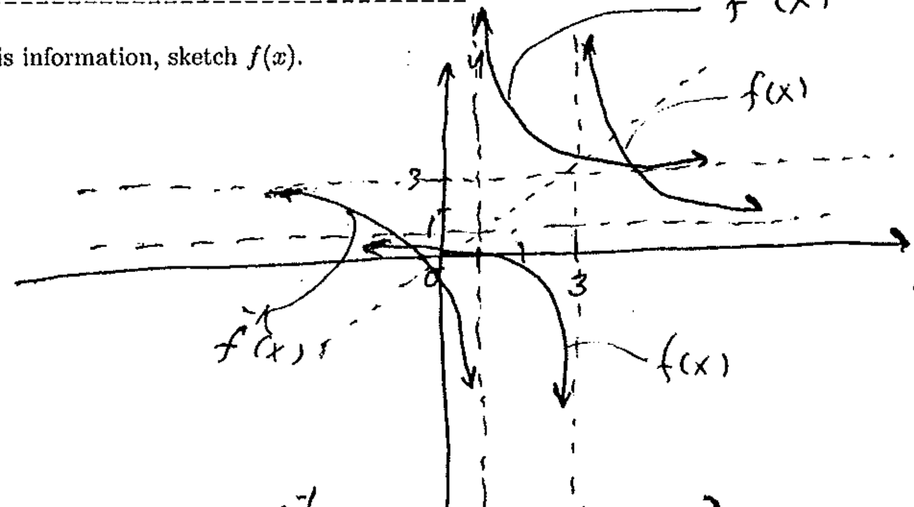
(b) Draw a signs table for $f(x)$

Factor		-2		3	
$x+2$	$-$	0	$+$		$+$
$x-3$	$-$		$-$	0	$+$
$f(x)$	$+$	0	$-$	0	$+$

(c) What is the behavior of $f(x)$ as x goes to $+\infty$ and $-\infty$?

$f(x) \to 1$ as $x \to -\infty$ or $x \to \infty$

(d) Using this information, sketch $f(x)$.



(e) Calculate the inverse of $f(x)$. $f^{-1}(x) = (3x+2)/(x-1)$

$y = \frac{x+2}{x-3}$

$y(x-3) = x+2$

$xy - x = 3y + 2$

$x(y-1) = 3y+2$

$x = (3y+2)/(y-1)$

$\Rightarrow f^{-1}(y) = \frac{3y+2}{y-1} \Rightarrow f^{-1}(x) = \frac{3x+2}{x-1}$

(f) Verify that $f^{-1}[f(x)] = x$.

$f^{-1}(f(x)) = f^{-1}\left(\frac{x+2}{x-3}\right) = \frac{\left[3\left(\frac{x+2}{x-3}\right) + 2\right] \cdot x-3}{\left[\left(\frac{x+2}{x-3}\right) - 1\right] \cdot x-3}$

$f^{-1}(f(x)) = \frac{3(x+2) + 2(x-3)}{x+2 - 1(x-3)} = \frac{5x}{5} = x$

(g) What is the domain of $f^{-1}(x)$? $\mathbb{R} - \{1\}$ or $(-\infty, 1) \cup (1, \infty)$

(h) Draw a signs table for $f^{-1}(x)$

Factor		$-2/3$		3	
$3x+2$	$-$	0	$+$		$+$
$x-1$	$-$		$-$	0	$+$
$f^{-1}(x)$	$+$	0	$-$	0	$+$

(i) What is the behavior of $f^{-1}(x)$ as x goes to $+\infty$ and $-\infty$?

$f^{-1}(x) \to 3$ as $x \to -\infty$ or $x \to \infty$

(j) On the same graph as the plot for $f(x)$, plot f^{-1}. Clearly mark which is which.

PROBLEM 3. HIGHER ORDER POLYNOMIAL Consider the higher order polynomial function $x^3 - 4x^2 - 5x$.

(a) What is the behavior near $+\infty$ and $-\infty$?
When x tends to $-\infty$, $f(x)$ goes to $-\infty$
When x tends to $+\infty$, $f(x)$ goes to $+\infty$

(b) Factor the function: $x(x^2 - 4x - 5) = x(x-5)(x+1)$

(c) Determine the $x-$ and $y-$ intercepts

$f(0) = 0^3 - 4 \cdot 0^2 - 5 \cdot 0 = 0$ y-intercept

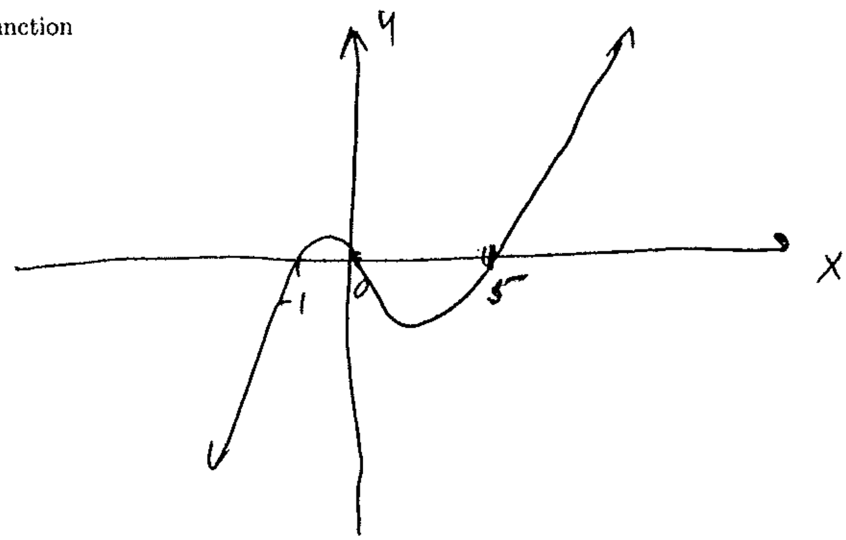
For x-intercepts set factors to zero and solve for x

$x-$intercept(s): $\{-1, 0, 5\}$ $y-$intercept: 0

(d) Draw a signs table

Factor		-1		0		5	
x	$-$		$-$	0	$+$		$+$
$x+1$	$-$	0	$+$		$+$		$+$
$x-5$	$-$		$-$		$-$	0	$+$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

(e) Sketch the function

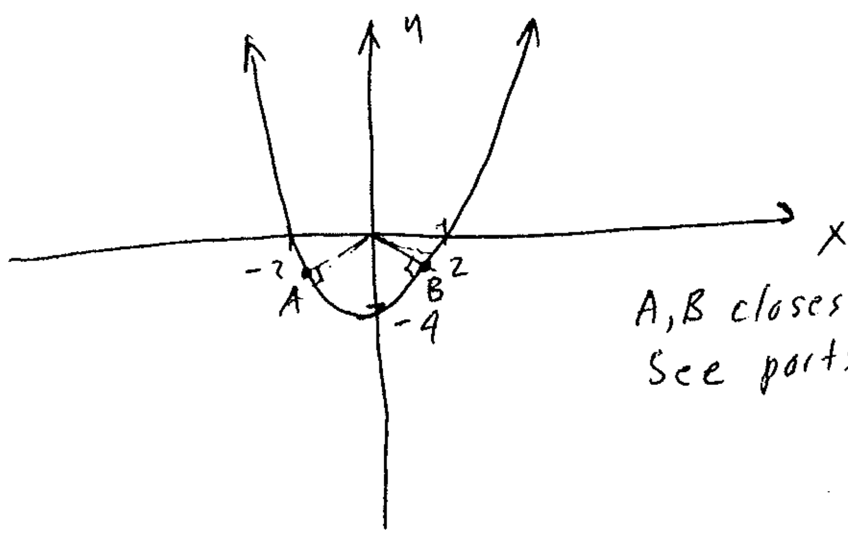


(f) Deduce the domain of definition of $\sqrt{x^3 - 4x^2 - 5x}$. Write the domain in interval notation.

$[-1, 0] \cup [5, \infty)$

PROBLEM 4. APPLIED PROBLEM. A near-Earth asteroid approaches the Earth in a parabolic orbit defined by the equation $y = x^2 - 4$ (the Earth is at the origin of the coordinate system, E(0,0)). The units used here are millions of miles.

(a) Sketch the parabolic orbit. Make sure to indicate the x-intercepts and the y-intercept. Draw the two points where the asteroid is the closest to the Earth.



A, B closest to earth
See parts (b) & (c)

(b) The asteroid is a point $A(x, y)$ on the parabola. What is its distance between the asteroid and the Earth as a function of x only?

$$D = \sqrt{x^2 + (x^2-4)^2}$$

$$D = \sqrt{x^4 - 7x^2 + 16}$$

(c) What is the smallest distance of approach between the two objects? Hint: doing the problem directly is hard! Instead, consider minimizing the square of the distance, as a function of the square of x. A change of variable will be useful).

$$D^2 = x^4 - 7x^2 + 16$$

$$t = x^2$$

$$D^2 = t^2 - 7t + 16$$

Use vertex formula $t = -\frac{b}{2a}$

$$t = \frac{-(-7)}{2(1)} = \frac{7}{2}$$

$$x^2 = 7/2 \qquad x = \pm\sqrt{7/2}$$

$$y = x^2 - 4 = \frac{7}{2} - 4$$

$$y = 7/2 - \frac{8}{2} = -1/2$$

$$D = \sqrt{7/2 + (-1/2)^2}$$

$$D = \sqrt{\frac{7}{2} + \frac{1}{4}} = \sqrt{\frac{14}{4} + \frac{1}{4}} = \sqrt{\frac{15}{4}}$$

$$D = \frac{\sqrt{15}}{2} \approx 1.94 \text{ million miles} \quad \underline{\text{ans}}$$

Note: Points A & B are $(-\sqrt{\frac{7}{2}}, -\frac{1}{2})$, $(\sqrt{7/2}, -1/2)$